TABLE 5.1 Sieve of Eratosthenes

	(2)	(3)	4	(5)	6	(7)	8	9	10	(11)	12	(13)	14
15	16	(17)	18	(19)	20	21	22	(23)	24	25	26	27	28
(29)	30	(31)	32	33	34	35	36	(37)	38	39	40	(41)	42
(43)	44	45	46	(47)	48	49	50	51	52	(53)	54	55	56
57	58	(59)	60	(61)	62	63	64	65	66	(67)	68	69	70
(71)	72	(73)	74	75	76	77	78	(79)	80	81	82	(83)	84
85	86	87	88	(89)	90	91	92	93	94	95	96	(97)	98
99	100												

Every circled number is prime. Theoretically, such a sieve can be constructed for any value of n.

Decide whether the following statements are true *or* false.

1. Every natural number is divisible by 1.

2. No natural number is both prime and composite.

3. There are no even prime numbers.

4. If n is a natural number and $9 \mid n$, then $3 \mid n$.

5. If n is a natural number and $5 \mid n$, then $10 \mid n$.

6. 1 is the smallest prime number.

7. Every natural number is both a factor and a multiple of itself.

8. If 16 divides a natural number, then 2, 4, and 8 must also divide that natural number.

9. The composite number 50 has exactly two prime factorizations.

10. The prime number 53 has exactly two natural number factors.

Find all natural number factors of each number.

11. 12 **12.** 18 **13.** 20 **14.** 28

15. 52 **16.** 63 **17.** 120 **18.** 172

Use divisibility tests to decide whether the given number is divisible by
(a) 2 **(b)** 3 **(c)** 4 **(d)** 5 **(e)** 6 **(f)** 8 **(g)** 9 **(h)** 10 **(i)** 12.

19. 315 **20.** 7,425 **21.** 1,092 **22.** 4,488 **23.** 630

24. 25,025 **25.** 45,815 **26.** 5,940 **27.** 123,456,789 **28.** 987,654,321

Selected Answers

1. true **3.** false **5.** false **7.** true **9.** false **11.** 1, 2, 3, 4, 6, 12 **13.** 1, 2, 4, 5, 10, 20 **15.** 1, 2, 4, 13, 26, 52 **17.** 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120 **19.** **(a)** no **(b)** yes **(c)** no **(d)** yes **(e)** no **(f)** no **(g)** yes **(h)** no **(i)** no **21.** **(a)** yes **(b)** yes **(c)** yes **(d)** no **(e)** yes **(f)** no **(g)** no **(h)** no **(i)** yes **23.** **(a)** yes **(b)** yes **(c)** no **(d)** yes **(e)** yes **(f)** no **(g)** yes **(h)** yes **(i)** no **25.** **(a)** no **(b)** no **(c)** no **(d)** yes **(e)** no **(f)** no **(g)** no **(h)** no **(i)** no **27.** **(a)** no **(b)** yes **(c)** no **(d)** no **(e)** no **(f)** no **(g)** yes **(h)** no **(i)** no **29.** 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199 **33.** The last four digits must form a number divisible by 16. The number 456,882,320 is divisible by 16, since 2,320 is divisible by 16: $2{,}320 = 16 \cdot 145$. **35.** $2^2 \cdot 3 \cdot 5^2$ **37.** $5^2 \cdot 17$ **39.** $3 \cdot 5 \cdot 59$ **41.** $3^2 \cdot 5^2 \cdot 7$ **43.** yes **45.** no **47.** yes **49.** no **51.** 840 **53.** 2,520 **57.** 0, 2, 4, 6, 8 **59.** 0, 4, 8 **61.** 0, 6 **63.** 6 **65.** 9 **67.** 12 **69.** 27 **71.** leap year **73.** not a leap year

Continue the Sieve of Eratosthenes in Table 5.1 from 101 to 200. (You need only check for divisibility by primes through 13.) List the primes between 100 and 200.

List two primes that are consecutive natural numbers. Can there be any others?

Can there be three primes that are consecutive natural numbers? Explain.

For a natural number to be divisible by both 2 and 5, what must be true about its last digit?

Consider the divisibility tests for 2, 4, and 8 (all powers of 2). Use inductive reasoning to predict the divisibility test for 16. Then, use the test to show that 456,882,320 is divisible by 16.

Find the prime factorization of each composite number.

240 **35.** 300 **36.** 360 **37.** 425

663 **39.** 885 **40.** 1,280 **41.** 1,575

Here is a divisibility test for 7.
Double the last digit of the given number, and subtract this value from the given number with the last digit omitted.
Repeat the process of part (a) as many times as necessary until the number obtained can easily be divided by 7.
If the final number obtained is divisible by 7, *then the given number is also divisible by* 7. *If the final number is not divisible by* 7, *then neither is the given number.*

Use this divisibility test to determine whether or not each of the following is divisible by 7.

142,891 **43.** 409,311 **44.** 458,485 **45.** 287,824

Here is a divisibility test for 11.
Starting at the left of the given number, add together every other digit.
Add together the remaining digits.
Subtract the smaller of the two sums from the larger.
If the final number obtained is divisible by 11, *then the given number is also divisible by* 11. *If the final number is not divisible by* 11, *then neither is the given number.*

Use this divisibility test to determine whether or not each of the following is divisible by 11.

8,493,969 **47.** 847,667,942 **48.** 453,896,248 **49.** 552,749,913

Consider the divisibility test for the composite number 6, and make a conjecture for the divisibility test for the composite number 15.

Find the smallest natural number that is divisible by all of the numbers in the group of numbers listed.

2, 3, 5, 7, 8 **52.** 2, 3, 4, 9, 10 **53.** 2, 3, 4, 5, 6, 7, 8, 9 **54.** 2, 3, 4, 5, 6, 7, 8, 9, 12

Explain why the answers in Exercises 53 and 54 must be the same.

Explain why the answer in Exercise 51 would not change if 2 were omitted from the group of numbers.

Determine all possible digit replacements for x so that the first number is divisible by the second. For example, $37{,}58x$ *is divisible by* 2 *if* $x = 0, 2, 4, 6,$ *or* 8.

398,87x; 2 **58.** 2,45x,765; 3 **59.** 64,537,84x; 4 **60.** 2,143,89x; 5

985,23x; 6 **62.** 7,643,24x; 8 **63.** 4,329,7x5; 9 **64.** 23,x54,470; 10

Common Factors and Multiples

Decide whether each of the following is true *or* false.

1. Two even natural numbers cannot be relatively prime.

2. Two different prime numbers must be relatively prime.

3. If p is a prime number, then the greatest common factor of p and p^2 is p.

4. If p is a prime number, then the least common multiple of p and p^2 is p^3.

5. There is no prime number p such that the greatest common factor of p and 2 is 2.

6. The set of all common multiples of two given natural numbers is finite.

7. Two natural numbers must have at least one common factor.

8. The least common multiple of two different primes is their product.

9. Two composite numbers may be relatively prime.

10. The set of all common factors of two given natural numbers is finite.

Use the prime factors method to find the greatest common factor of each group of numbers.

11. 70 and 120 **12.** 180 and 300 **13.** 480 and 1,800

14. 168 and 504 **15.** 28, 35, and 56 **16.** 252, 308, and 504

Use the method of dividing by prime factors to find the greatest common factor of each group of numbers.

17. 60 and 84 **18.** 130 and 455 **19.** 310 and 460

20. 234 and 470 **21.** 12, 18, and 30 **22.** 450, 1,500, and 432

Use the Euclidean algorithm to find the greatest common factor of each group of numbers.

23. 36 and 60 **24.** 25 and 70 **25.** 84 and 180

26. 72 and 120 **27.** 210 and 560 **28.** 150 and 480

29. Explain in your own words how to find the greatest common factor of a group of numbers.

30. Explain in your own words how to find the least common multiple of a group of numbers.

Use the prime factors method to find the least common multiple of each group of numbers.

31. 24 and 30 **32.** 12 and 32 **33.** 56 and 96

34. 28 and 70 **35.** 30, 40, and 70 **36.** 24, 36, and 48

Use the formula given in the text and the results of Exercises 23–28 to find the least common multiple of each group of numbers.

37. 36 and 60 **38.** 25 and 70 **39.** 84 and 180

40. 72 and 120 **41.** 210 and 560 **42.** 150 and 480

43. If p, q, and r are different primes, and a, b, and c are natural numbers such that $a > b > c$,
(a) what is the greatest common factor of $p^a q^c r^b$ and $p^b q^a r^c$?
(b) what is the least common multiple of $p^b q^a$, $q^b r^c$, and $p^a r^b$?

44. Find **(a)** the greatest common factor and **(b)** the least common multiple of $2^{31} \cdot 5^{17} \cdot 7^{21}$ and $2^{34} \cdot 5^{22} \cdot 7^{13}$. Leave your answers in prime factored form.

Word Problems

1. What is the largest size of square tiles that can be used to cover the floor of a rectangular room 2.56 by 1.44 m?
2. The tables in the school canteen are arranged in the same number of tables in each row. If they can be arranged in 4 rows, 5 rows, or 6 rows, what is the least number of tables in the canteen?
3. A mountain chalet has equal capacity rooms. It can accommodate 78 guest on 1st floor, 54 on 2nd floor, 84 on 3rd floor and 48 on the top floor. What's the maximum number of beds in each room and what's the number of rooms in the chalet?
4. A florist has 72 white roses and 96 red roses. How many bouquets can she make at most, provided that they all have equal number of red roses and all have equal number of white roses?
5. This table lists the numbers of teeth on a five-speed bicycle gears. Suppose one tooth on each gear is marked. How many pedal gear revolutions will it take for the marked teeth to click into each other again?

	1st Gear	2nd Gear	3rd Gear	4th Gear	5th Gear
Pedal Gear	52	52	52	52	52
Axle Gear	28	24	20	17	14

6. You need to make shelves of these lengths: 60 cm, 75 cm, or 50 cm. What is the most efficient length of wooden planks to use?
7. You have 842 rectangular 20 × 30 cm tiles. How many can be used to tile the largest square possible unless they are cut?
8. Five tram lines run in these intervals: 5, 8, 10, 12 and 15 min. They all set off at noon. When will they all meet again?
9. There are less than 100 apples in a basket. If they are counted by 3's, there are two left over; by 4's, there are three left over and by 5's, there are four left over. How many apples are in the basket?
10. There are less than 500 sheep in the shepherd's flock. If he leads them by 2's, 3's, 4's, 5's or 6's, one sheep always remains. If he leads them by 7's, none is left alone. How many sheep has he got?
11. When members of a choir stand by 3's, one remains. If they stand by 4's, three are left. How many members should join them, so that they can stand in 3 and 4 rows?
12. A toddler built pyramids from cubes. When he was building 3-cube pyramids, 1 cube was left. When building 5-cube pyramids, he had 6 pyramids less than before and 3 cubes left. What's the number of cubes?

Selected Answers

1. true **3.** true **5.** false **7.** true **9.** true **11.** 10 **13.** 120 **15.** 7 **17.** 12 **19.** 10
21. 6 **23.** 12 **25.** 12 **27.** 70 **31.** 120 **33.** 672 **35.** 840 **37.** 180 **39.** 1,260 **41.** 1,680
43. **(a)** $p^b q^c r^c$ **(b)** $p^a q^a r^b$ **45.** 30 **47.** 15 **49.** 2,880 **51.** **(a)** 6 **(b)** 36 **53.** **(a)** 18 **(b)** 216

1. 16 cm
2. LCM (4,5,6) = 60
3. GCF (78,54,84,48) = 6; 44 rooms
4. GCF (72,96) = 24
5. No. of revolutions are 7, 6, 5, 17, 7
6. 3 m
7. 726
8. at 2 p.m.
9. 59 apples
10. 301 sheep
11. 5 members
12. 43 cubes