Number Sets

Graph each set on a number line.

1. $\{2, 3, 4, 5\}$

2. $\{0, 2, 4, 6, 8\}$

3. $\{-4, -3, -2, -1, 0, 1\}$

4. $\{-6, -5, -4, -3, -2\}$

5. $\left\{-\frac{1}{2}, \frac{3}{4}, \frac{5}{3}, \frac{7}{2}\right\}$

6. $\left\{-\frac{3}{5}, -\frac{1}{10}, \frac{9}{8}, \frac{12}{5}, \frac{13}{4}\right\}$

List the numbers in the given set that belong to **(a)** *the natural numbers,* **(b)** *the whole numbers,* **(c)** *the integers,* **(d)** *the rational numbers,* **(e)** *the irrational numbers, and* **(f)** *the real numbers.*

7. $\left\{-9, -\sqrt{7}, -\frac{5}{4}, -\frac{3}{5}, 0, \sqrt{5}, 3, 5.9, 7\right\}$

8. $\left\{-5.3, -5, -\sqrt{3}, -1, -\frac{1}{9}, 0, 1.2, 1.8, 3, \sqrt{11}\right\}$

Decide whether each of the following statements is true *or* false.

9. $-2 < -1$

10. $-8 < -4$

11. $-3 \geq -7$

12. $-9 \geq -12$

13. $-15 \leq -20$

14. $-21 \leq -27$

15. $-8 \leq -(-4)$

16. $-9 \leq -(-6)$

17. $0 \leq -(-4)$

18. $0 \geq -(-6)$

19. $6 > -(-2)$

20. $-8 > -(-2)$

Give **(a)** *the additive inverse and* **(b)** *the absolute value of each of the following.*

21. 5

22. 9

23. -6

24. -8

25. A statement commonly heard is "Absolute value is always positive." Is this true? If not, explain.

26. If a is a negative number, then is $-|-a|$ positive or negative?

27. Fill in the blanks with the correct values: The opposite of -3 is ——, while the absolute value of -3 is ——. The additive inverse of -3 is ——, while the additive inverse of the absolute value of -3 is ——.

28. True or false: For all real numbers a and b, $|a - b| = |b - a|$.

Specify each set by listing its elements. If there are n elements, write $\emptyset$.

29. $\{x \mid x$ is a natural number less than 7$\}$

30. $\{m \mid m$ is a whole number less than 9$\}$

31. $\{a \mid a$ is an even integer greater than 10$\}$

32. $\{k \mid k$ is a natural number less than 1$\}$

Tell whether each statement is true *or* false.

45. Every rational number is an integer.

46. Every natural number is an integer.

47. Every integer is a rational number.

48. Every whole number is a real number.

49. Some rational numbers are irrational.

50. Some natural numbers are whole numbers.

51. Some rational numbers are integers.

52. Some real numbers are integers.

53. Every rational number is a real number.

54. Some integers are not real numbers.

55. Every integer is positive.

56. Every whole number is positive.

57. Some irrational numbers are negative.

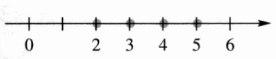
58. Some real numbers are not rational.

59. Not every rational number is positive.

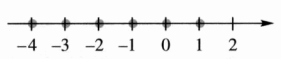
60. Some whole numbers are not integers.

Selected Answers

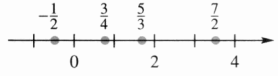
1.



3.



5.



7. **(a)** 3, 7 **(b)** 0, 3, 7 **(c)** $-9, 0, 3, 7$ **(d)** $-9, -5/4, -3/5, 0, 3, 5.9, 7$ **(e)** $-\sqrt{7}, \sqrt{5}$ **(f)** All are real numbers.
9. true **11.** true **13.** false **15.** true **17.** true **19.** true **21.** **(a)** -5 **(b)** 5 **23.** **(a)** 6 **(b)** 6
27. 3; 3; 3; -3 **29.** $\{1, 2, 3, 4, 5, 6\}$ **31.** $\{12, 14, 16, 18, \ldots\}$ **33.** $\emptyset$ **35.** $\{3, -3\}$
45. F **47.** T **49.** F **51.** T **53.** T **55.** F **57.** T **59.** T

Fractions, Decimals and Surds

Identify each of the following as rational *or* irrational.

1. $\frac{4}{7}$

2. $\frac{5}{8}$

3. $\sqrt{6}$

4. $\sqrt{13}$

5. .89

6. .76

7. $.\overline{89}$

8. $.\overline{76}$

9. .878778777877778. . .

10. .434334333433334. . .

11. 3.14159

12. $\frac{22}{7}$

13. π

14. 0

15. **(a)** Find the following sum:

$$\begin{array}{r} .272772777277772\ldots \\ +\ .616116111611116\ldots \\ \hline \end{array}$$

(b) Based on the result of part (a), we can conclude that the sum of two —— numbers may be a(n) —— number.

16. **(a)** Find the following sum:

$$\begin{array}{r} .010110111011110\ldots \\ +\ .252552555255552\ldots \\ \hline \end{array}$$

(b) Based on the result of part (a), we can conclude that the sum of two —— numbers may be a(n) —— number.

Use a calculator to find a rational decimal approximation for each of the following irrational numbers. Give as many places as your calculator shows.

17. $\sqrt{39}$

18. $\sqrt{44}$

19. $\sqrt{15.1}$

20. $\sqrt{33.6}$

21. $\sqrt{884}$

22. $\sqrt{643}$

23. $\sqrt{\frac{9}{8}}$

24. $\sqrt{\frac{6}{5}}$

Use the methods of Examples 2 and 3 to simplify each of the following expressions. Then, use a calculator to approximate both the given expression and the simplified expression. (Both should be the same.)

39. $\sqrt{50}$ **40.** $\sqrt{32}$ **41.** $\sqrt{75}$ **42.** $\sqrt{150}$ **43.** $\sqrt{288}$ **44.** $\sqrt{200}$ **45.** $\frac{5}{\sqrt{6}}$ **46.** $\frac{3}{\sqrt{2}}$

47. $\sqrt{\frac{7}{4}}$

48. $\sqrt{\frac{8}{9}}$

49. $\sqrt{\frac{7}{3}}$

50. $\sqrt{\frac{14}{5}}$

Use the method of Example 4 to perform the indicated operations.

53. $\sqrt{6} + \sqrt{6}$ **54.** $\sqrt{11} + \sqrt{11}$ **55.** $\sqrt{17} + 2\sqrt{17}$ **56.** $3\sqrt{19} + \sqrt{19}$
57. $5\sqrt{7} - \sqrt{7}$ **58.** $3\sqrt{27} - \sqrt{27}$ **59.** $3\sqrt{18} + \sqrt{2}$ **60.** $2\sqrt{48} - \sqrt{3}$
61. $-\sqrt{12} + \sqrt{75}$ **62.** $2\sqrt{27} - \sqrt{300}$ **63.** $5\sqrt{72} - 2\sqrt{50}$ **64.** $6\sqrt{18} - 4\sqrt{32}$

Find the rational number halfway between the two given rational numbers.

75. $\frac{1}{2}, \frac{3}{4}$

76. $\frac{1}{3}, \frac{5}{12}$

77. $\frac{3}{5}, \frac{2}{3}$

78. $\frac{7}{12}, \frac{5}{8}$

79. $-\frac{2}{3}, -\frac{5}{6}$

80. $-3, -\frac{5}{2}$

Use McKay's theorem *to find a rational number between the two given rational numbers.*

81. $\frac{5}{6}$ and $\frac{9}{13}$ **82.** $\frac{10}{11}$ and $\frac{13}{19}$ **83.** $\frac{4}{13}$ and $\frac{9}{16}$

84. $\frac{6}{11}$ and $\frac{8}{9}$ **85.** 2 and 3 **86.** 3 and 4

87. Apply McKay's theorem to any pair of consecutive integers, and make a conjecture about what happens in this case.

88. Explain in your own words how to find the rational number that is one-fourth of the way between two different rational numbers.

Convert each rational number into either a repeating or a terminating decimal. Use a calculator if your instructor so allows.

89. $\frac{3}{4}$ **90.** $\frac{7}{8}$ **91.** $\frac{3}{16}$ **92.** $\frac{9}{32}$

93. $\frac{3}{11}$ **94.** $\frac{9}{11}$ **95.** $\frac{2}{7}$ **96.** $\frac{11}{15}$

Convert each terminating decimal into a quotient of integers. Write each in lowest terms.

97. .4 **98.** .9 **99.** .85 **100.** .105 **101.** .934 **102.** .7984

Convert each repeating decimal into a quotient of integers. Write each in lowest terms.

103. $.\overline{8}$ **104.** $.\overline{1}$ **105.** $.\overline{54}$ **106.** $.\overline{36}$

107. $.4\overline{3}$ **108.** $.2\overline{6}$ **109.** $1.\overline{9}$ **110.** $3.0\overline{9}$

Use the method of Example 7 to decide whether each of the following rational numbers would yield a repeating or a terminating decimal. (Hint: Write in lowest terms before trying to decide.)

111. $\frac{8}{15}$ **112.** $\frac{8}{35}$ **113.** $\frac{13}{125}$ **114.** $\frac{3}{24}$ **115.** $\frac{22}{55}$ **116.** $\frac{24}{75}$

Selected Answers

1. rational **3.** irrational **5.** rational **7.** rational **9.** irrational **11.** rational **13.** irrational
15. **(a)** $.\overline{8}$ **(b)** irrational, rational

The number of digits shown will vary among calculator models in Exercises 17–23.
17. 6.244997998 **19.** 3.885871846 **21.** 29.73213749 **23.** 1.060660172 **25.** yes, yes
39. $5\sqrt{2}$; 7.071067812 **41.** $5\sqrt{3}$; 8.660254038 **43.** $12\sqrt{2}$; 16.97056275 **45.** $5\sqrt{6}/6$; 2.041241452
47. $\sqrt{7}/2$; 1.322875656 **49.** $\sqrt{21}/3$ 1.527525232 **53.** $2\sqrt{6}$ **55.** $3\sqrt{17}$ **57.** $4\sqrt{7}$ **59.** $10\sqrt{2}$
61. $3\sqrt{3}$ **63.** $20\sqrt{2}$ **65.** **(a)** 1.414213562 **(b)** 2.645751311 **(c)** 3.633180425 **(d)** 5 **67.** **(a)** $\sqrt[3]{a}$
77. 19/30 **79.** –3/4 **81.** 14/19 **83.** 13/29 **85.** 5/2 **87.** It gives the rational number halfway between the two integers. **89.** .75 **91.** .1875 **93.** $.\overline{27}$ **95.** $.\overline{285714}$ **97.** 2/5 **99.** 17/20 **101.** 467/500
103. 8/9 **105.** 6/11 **107.** 13/30 **109.** 2 **111.** repeating **113.** terminating **115.** terminating

31. **(a)** 78.4 **(b)** 78.41 **33.** **(a)** .1 **(b)** .08 **35.** **(a)** 12.7 **(b)** 12.69 **37.** 42% **39.** 36.5%
41. .8% **43.** 210% **45.** 20% **47.** 1% **49.** 37 1/2% **51.** 150% **55.** **(a)** 5 **(b)** 24 **(c)** 8 **(d)** .5
1. true **3.** false **5.** false **7.** false **9.** false

Percents and Decimals

Decide whether each of the following is true *or* false.

1. 50% of a quantity is the same as 1/2 of the quantity.

2. 200% of 8 is 16.

3. When 435.67 is rounded to the nearest ten, the answer is 435.7.

4. When 668.342 is rounded to the nearest hundredth, the answer is 668.34.

5. A football team that wins 10 games and loses 6 games has a winning percentage of 60%.

6. To find 25% of a quantity, we may simply divide the quantity by 4.

Round each of the following numbers to the nearest **(a)** *tenth;* **(b)** *hundredth. Always round from the original number.*

31. 78.414 **32.** 3,689.537 **33.** .0837
34. .0658 **35.** 12.68925 **36.** 43.99613

Convert each decimal to a percent.

37. .42 **38.** .87 **39.** .365 **40.** .792
41. .008 **42.** .0093 **43.** 2.1 **44.** 8.9

Convert each fraction to a percent.

45. $\frac{1}{5}$ **46.** $\frac{2}{5}$ **47.** $\frac{1}{100}$ **48.** $\frac{1}{50}$

49. $\frac{3}{8}$ **50.** $\frac{5}{6}$ **51.** $\frac{3}{2}$ **52.** $\frac{7}{4}$

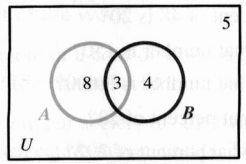
53. Explain the difference between 1/2 of a quantity and 1/2% of the quantity.

54. In the left column of the chart below there are some common percents, found in many everyday situations. In the right column are fractional equivalents of these percents. Match the fractions in the right column with their equivalent percents in the left column.

(a) 25%	**A.** $\frac{1}{3}$
(b) 10%	**B.** $\frac{1}{50}$
(c) 2%	**C.** $\frac{3}{4}$
(d) 20%	**D.** $\frac{1}{10}$
(e) 75%	**E.** $\frac{1}{4}$
(f) $33\frac{1}{3}\%$	**F.** $\frac{1}{5}$

55. Fill in each blank with the appropriate numerical response.
(a) 5% means ____ in every 100.
(b) 25% means 6 in every ____.
(c) 200% means ____ for every 4.
(d) .5% means ____ in every 100.
(e) ____% means 12 for every 2.

56. The following Venn diagram shows the number of elements in the four regions formed.



(a) What percent of the elements in the universe are in $A \cap B$?
(b) What percent of the elements in the universe are in A but not in B?
(c) What percent of the elements in $A \cup B$ are in $A \cap B$?
(d) What percent of the elements in the universe are in neither A nor B?