Scientific Notation

13. Which one of the following is equal to 1 $(a \neq 0)$?

(a) $3a^0$ (b) $-3a^0$ (c) $(3a)^0$ (d) $3(-a)^0$

14. Which one of the following represents a negative number?

(a) $(-3)^{-2}$ (b) $(-1,000)^0$ (c) $(-4)^0 - (-3)^0$ (d) $(-5)^{-3}$

Evaluate each exponential expression.

15. 7^{-2} **16.** 4^{-1} **17.** -7^{-2} **18.** -4^{-1} **19.** $\dfrac{2}{(-4)^{-3}}$

20. $\dfrac{2^{-3}}{3^{-2}}$ **21.** $\dfrac{5^{-1}}{4^{-2}}$ **22.** $\left(\dfrac{1}{2}\right)^{-3}$ **23.** $\left(\dfrac{1}{5}\right)^{-3}$ **24.** $\left(\dfrac{2}{3}\right)^{-2}$

25. $\left(\dfrac{4}{5}\right)^{-2}$ **26.** $3^{-1} + 2^{-1}$ **27.** $4^{-1} + 5^{-1}$ **28.** 8^0 **29.** 12^0

30. $(-23)^0$ **31.** $(-4)^0$ **32.** -2^0 **33.** $3^0 - 4^0$ **34.** $-8^0 - 7^0$

35. In order to raise a fraction to a negative power, we may change the fraction to its _____ and change the exponent to the _____ of the original exponent.

36. Explain in your own words how we raise a power to a power.

37. Which one of the following is correct?

(a) $-\dfrac{3}{4} = \left(\dfrac{3}{4}\right)^{-1}$ (b) $\dfrac{3^{-1}}{4^{-1}} = \left(\dfrac{4}{3}\right)^{-1}$ (c) $\dfrac{3^{-1}}{4} = \dfrac{3}{4^{-1}}$ (d) $\dfrac{3^{-1}}{4^{-1}} = \left(\dfrac{3}{4}\right)^{-1}$

38. Which one of the following is incorrect?

(a) $(3r)^{-2} = 3^{-2}r^{-2}$ (b) $3r^{-2} = (3r)^{-2}$ (c) $(3r)^{-2} = \dfrac{1}{(3r)^2}$ (d) $(3r)^{-2} = \dfrac{r^{-2}}{9}$

Use the product, quotient, and power rules to simplify each expression. Write answers with only positive exponents. Assume that all variables represent nonzero real numbers.

39. $x^{12} \cdot x^4$ **40.** $\dfrac{x^{12}}{x^4}$ **41.** $\dfrac{5^{17}}{5^{16}}$ **42.** $\dfrac{3^{12}}{3^{13}}$

43. $\dfrac{3^{-5}}{3^{-2}}$ **44.** $\dfrac{2^{-4}}{2^{-3}}$ **45.** $\dfrac{9^{-1}}{9}$ **46.** $\dfrac{12}{12^{-1}}$

47. $t^5 t^{-12}$ **48.** $p^5 p^{-6}$ **49.** $(3x)^2$ **50.** $(-2x^{-2})^2$

51. $a^{-3}a^2a^{-4}$ **52.** $k^{-5}k^{-3}k^4$ **53.** $\dfrac{x^7}{x^{-4}}$ **54.** $\dfrac{p^{-3}}{p^5}$

55. $\dfrac{r^3 r^{-4}}{r^{-2}r^{-5}}$ **56.** $\dfrac{z^{-4}z^{-2}}{z^3 z^{-1}}$ **57.** $7k^2(-2k)(4k^{-5})$ **58.** $3a^2(-5a^{-6})(-2a)$

59. $(z^3)^{-2}z^2$ **60.** $(p^{-1})^3 p^{-4}$ **61.** $-3r^{-1}(r^{-3})^2$ **62.** $2(y^{-3})^4(y^6)$

63. $(3a^{-2})^3(a^3)^{-4}$ **64.** $(m^5)^{-2}(3m^{-2})^3$ **65.** $(x^{-5}y^2)^{-1}$ **66.** $(a^{-3}b^{-5})^2$

67. $(2p^2q^{-3})^2(4p^{-3}q)^2$ **68.** $(-5y^2z^{-4})^2(2yz^5)^{-3}$ **69.** $\dfrac{(p^{-2})^3}{5p^4}$ **70.** $\dfrac{(m^4)^{-1}}{9m^3}$

71. $\dfrac{4a^5(a^{-1})^3}{(a^{-2})^{-2}}$ **72.** $\dfrac{12k^{-2}(k^{-3})^{-4}}{6k^5}$ **73.** $\dfrac{(-y^{-4})^2}{6(y^{-5})^{-1}}$ **74.** $\dfrac{2(-m^{-1})^{-4}}{9(m^{-3})^2}$

75. $\dfrac{(2k)^2 m^{-5}}{(km)^{-3}}$ **76.** $\dfrac{(3rs)^{-2}}{3^2 r^2 s^{-4}}$

Many students believe that the pairs of expressions shown in Exercises 77–79 represent the same quantity. This is wrong. Show that each expression in the pair represents a different quantity by replacing x with 2 and y with 3.

77. $(x+y)^{-1}$; $x^{-1} + y^{-1}$ **78.** $(x+y)^2$; $x^2 + y^2$ **79.** $(x^{-1} + y^{-1})^{-1}$; $x + y$

80. Which one of the following does not represent the reciprocal of x $(x \neq 0)$?

(a) x^{-1} (b) $\dfrac{1}{x}$ (c) $\left(\dfrac{1}{x^{-1}}\right)^{-1}$ (d) $-x$

Write each number in scientific notation.

81. 230 **82.** $46,500$ **83.** $.02$ **84.** $.0051$

Write each number without scientific notation.

85. 6.5×10^3 **86.** 2.317×10^5 **87.** 1.52×10^{-2} **88.** 1.63×10^{-4}

Use scientific notation to perform each of the following computations.

89. $\dfrac{.002 \times 3,900}{.000013}$ **90.** $\dfrac{.009 \times 600}{.02}$ **91.** $\dfrac{.0004 \times 56,000}{.000112}$

92. $\dfrac{.018 \times 20,000}{300 \times .0004}$ **93.** $\dfrac{840,000 \times .03}{.00021 \times 600}$ **94.** $\dfrac{28 \times .0045}{140 \times 1,500}$

Use scientific notation to work the following problems. Use a calculator as necessary.

95. The distance to the sun is 9.3×10^7 mi. How long would it take a rocket, traveling at 2.9×10^3 mph, to reach the sun?

96. A *light-year* is the distance that light travels in one year. Find the number of miles in a light-year if light travels 1.86×10^5 mi per second.

97. Use the information given in the previous two exercises to find the number of minutes necessary for light from the sun to reach the earth.

98. A computer can execute one addition in 1.4×10^{-7} seconds. How long would it take the computer to execute a trillion (10^{12}) additions? Give the answer in seconds and then in hours.

99. The planet Mercury has a mean distance from the sun of 3.6×10^7 mi, while the mean distance of Venus from the sun is 6.7×10^7 mi. How long would it take a spacecraft traveling at 1.55×10^3 mph to travel the distance represented by the difference of these two planets' mean distances from the sun?

100. When the distance between the centers of the moon and the earth is 4.60×10^8 m, an object on the line joining the centers of the moon and the earth exerts the same gravitational force on each when it is 4.14×10^8 m from the center of the earth. How far is the object from the center of the moon at that point?

101. Assume that the volume of the earth is 5×10^{14} m^3 and that the volume of a bacterium is 2.5×10^{-16} m^3. If the earth could be packed full of bacteria, how many would it contain?

102. Our galaxy is approximately 1.2×10^{17} km across. Suppose a spaceship could travel at 1.5×10^5 km per second (half the speed of light). Find the approximate number of years needed for the spaceship to cross the galaxy.

Selected Answers

1. 625 **3.** 25/9 **5.** -32 **7.** -8 **9.** -81 **13.** (c) **15.** 1/49 **17.** $-1/49$ **19.** -128
21. 16/5 **23.** 125 **25.** 25/16 **27.** 9/20 **29.** 1 **31.** 1 **33.** 0 **35.** reciprocal; additive inverse **37.** (d) **39.** x^{16} **41.** 5 **43.** 1/27 **45.** 1/81 **47.** $1/t^7$ **49.** $9x^2$ **51.** $1/a^5$
53. x^{11} **55.** r^6 **57.** $-56/k^2$ **59.** $1/z^4$ **61.** $-3/r^7$ **63.** $27/a^{18}$ **65.** x^5/y^2 **67.** $64/(p^2 q^4)$
69. $1/(5p^{10})$ **71.** $4/a^2$ **73.** $1/(6y^{13})$ **75.** $4k^5/m^2$ **77.** 1/5; 5/6; $1/5 \neq 5/6$ **79.** 6/5; 5; $6/5 \neq 5$
81. 2.3×10^2 **83.** 2×10^{-2} **85.** 6,500 **87.** .0152 **89.** 6×10^5 or 600,000 **91.** 2×10^5 or 200,000
93. 2×10^5 or 200,000 **95.** 3.2×10^4 hr **97.** 8.3 min **99.** 2×10^4 hr **101.** 2×10^{30} bacteria