Summary

Symbols Used in this Chapter

Connectives	Symbols	Types of Statements
and	$\wedge$	Conjunction
or	$\vee$	Disjunction
not	~	Negation
if . . . then	$\rightarrow$	Conditional
if and only if	$\leftrightarrow$	Biconditional
is equivalent to	$\equiv$	Equivalent

3.1 Statements and Quantifiers

Universal Quantifiers all, each, every, no(ne)

Existential Quantifiers some, there exists, (for) at least one

Negations of Quantified Statements

Statement	Negation
All do.	Some do not. (Equivalently: Not all do.)
Some do.	None do. (Equivalently: All do not.)

3.2 Truth Tables

Truth Tables for Negation, Conjunction, and Disjunction

p	$\sim p$
T	F
F	T

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

De Morgan's Laws

For any statements p and q,

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q.$$

Two statements are equivalent if they have the same truth value in *every* possible situation.

A logical statement having n component statements will have 2^n lines in its truth table.

3.3 The Conditional

Truth Table for the Conditional *if p, then q*

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

A statement that has all Ts in the final column completed in its truth table is a **tautology.**

Negation of $p \rightarrow q$ $p \wedge \sim q$

The disjunction $\sim p \vee q$ is equivalent to $p \rightarrow q$.

3.4 More on the Conditional

Statements Related to the Conditional

Direct statement	$p \rightarrow q$	(If p, then q.)
Converse	$q \rightarrow p$	(If q, then p.)
Inverse	$\sim p \rightarrow \sim q$	(If not p, then not q.)
Contrapositive	$\sim q \rightarrow \sim p$	(If not q, then not p.)

Various Translations of $p \rightarrow q$

The conditional $p \rightarrow q$ can be translated in any of the following ways.

If p, then q.	p is sufficient for q.
If p, q.	q is necessary for p.
p implies q.	All p's are q's.
p only if q.	q if p.

Truth Table for the Biconditional *p if and only if q*

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Negations of $p \leftrightarrow q$

The biconditional has three equivalent negations:

$\sim p \leftrightarrow q \equiv p \leftrightarrow \sim q \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Proofs

Direct Proof

To prove $p \rightarrow q$, we make a chain of obvious implications $p \rightarrow r \rightarrow s \rightarrow ... \rightarrow q$

Proof by Contrapositive

To prove $p \rightarrow q$, we prove its contrapositive $\sim q \rightarrow \sim p$. Since it is equivalent with the direct statement, it is valid.

Proof by Contradiction

To prove a statement by contradiction, start by assuming the opposite of what you would like to prove. Then show that the consequences of this premise are impossible (lead to a contradiciton). This means that your original statement must be true.