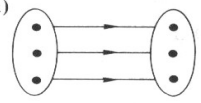
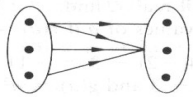
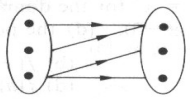
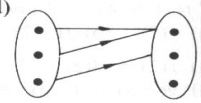
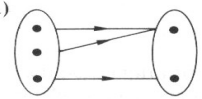
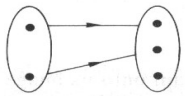
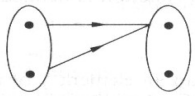
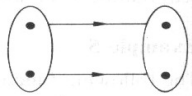
Functions

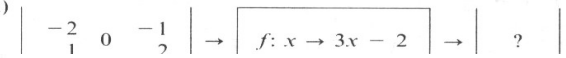
1. State which of the following arrow diagrams show functions.

(a)  (b)  (c)  (d) 

2. State which of the following arrow diagrams show: (i) a one-to-one function mapping into the co-domain, (ii) a one-to-one function mapping onto the co-domain, (iii) a many-to-one function mapping into the co-domain.

(a)  (b)  (c)  (d) 

3. State the range of each of the following 'function machines' for the domains shown.

(a) 

4. Draw arrow diagrams for the functions $f: x \to x + 2$, $g: x \to x^2 + 1$ and $h: x \to (x + 1)^2$ for the domain $\{-2, -1, 0, 1, 2\}$ and state the range of each function for this domain.

5. Draw arrow diagrams for the functions $f: x \to |x|$, $g: x \to |x| - 1$ and $h: x \to |x - 1|$ for the domain $\{-2, -1, 0, 1, 2\}$ and state the range of each function for this domain.

6. If $f(x) = 2x + 3$ find

(a) $f(2)$, (b) $f(-1)$, (c) $f(6)$, (d) the value of a if $f(a) = a$.

7. If $g(x) = x^2 - 6$ find

(a) $g(4)$, (b) $g(-4)$, (c) $g(2)$, (d) the possible values of a if $g(a) = a$.

8. If $f(x) = 2x^2$ and $g(x) = 3 - x$ find

(a) $f(3)$, (b) $f(-3)$, (c) $g(-3)$, (d) the possible values of a if $f(a) = g(a)$.

9. The function f is given by $f(x) = ax + b$. If $f(3) = 3$ and $f(4) = 5$, find a and b.

10. The function g is given by $g(x) = ax^2 - b$. If $g(2) = 5$ and $g(-1) = 2$, find the values of a and b and hence find $g(-4)$.

11. Each of the following functions maps an element x of the domain onto its image y, i.e. $f(x) = y$. Find the range of each function for the given domains and state whether the function is one-to-one or many-to-one.

(a) $f: x \to x + 3$ with domain $\{x: 0 \leqslant x \leqslant 4\}$,
(b) $f: x \to x - 2$ with domain $\{x: 0 \leqslant x \leqslant 4\}$,
(c) $f: x \to 2x$ with domain $\{x: 0 \leqslant x \leqslant 3\}$,
(d) $f: x \to 2x$ with domain $\{x: -3 \leqslant x \leqslant 3\}$,
(e) $f: x \to x^2$ with domain $\{x: -3 \leqslant x \leqslant 3\}$,
(f) $f: x \to \sqrt{x}$ with domain $\{x: 0 \leqslant x \leqslant 25\}$,
(g) $f: x \to |x|$ with domain $\{x: -3 \leqslant x \leqslant 3\}$,
(h) $f: x \to x^2$ with domain $\mathbb{R}$,
(i) $f: x \to |x|$ with domain $\mathbb{R}$,
(j) $f: x \to \dfrac{1}{x}$ with domain $\{x: x \geqslant 1\}$,
(k) $f: x \to x^2 + 4$ with domain $\mathbb{R}$,
(l) $f: x \to \dfrac{1}{x - 1}$ with domain $\{x \in \mathbb{R}: x \neq 1\}$.

12. The following functions map an element x of the domain onto its image y, i.e. $f: x \to y$. For each function state

(i) the domain for which the function is defined,

Answers

1. (a) and (d) **2.** (i) (b); (ii) (d); (iii) (c) **3.** (a) $\{-8, -5, -2, 1, 4\}$, (b) $\{-1, 1, 7\}$
4. $\{0, 1, 2, 3, 4\}$, $\{1, 2, 5\}$, $\{0, 1, 4, 9\}$ **5.** $\{0, 1, 2\}$, $\{-1, 0, 1\}$, $\{0, 1, 2, 3\}$
6. (a) 7 (b) 1 (c) 15 (d) -3 **7.** (a) 10 (b) 10 (c) -2 (d) 3 or -2
8. (a) 18 (b) 18 (c) 6 (d) $-1\frac{1}{2}$, 1 **9.** 2, -3 **10.** 1, -1, 17
11. (a) $\{y: 3 \leqslant y \leqslant 7\}$ one-to-one (b) $\{y: |y| \leqslant 2\}$ one-to-one
(c) $\{y: 0 \leqslant y \leqslant 6\}$ one-to-one (d) $\{y: |y| \leqslant 6\}$ one-to-one
(e) $\{y: 0 \leqslant y \leqslant 9\}$ many-to-one (f) $\{y: 0 \leqslant y \leqslant 5\}$ one-to-one
(g) $\{y: 0 \leqslant y \leqslant 3\}$ many-to-one (h) $\{y: y \geqslant 0\}$ many-to-one
(i) $\{y: y \geqslant 0\}$ many-to-one (j) $\{y: 0 < y \leqslant 1\}$ one-to-one
(k) $\{y: y \geqslant 4\}$ many-to-one (l) $\{y \in \mathbb{R}: y \neq 0\}$ one-to-one
12. (a) (i) $\mathbb{R}$ (ii) $\mathbb{R}$ (b) (i) $\mathbb{R}$ (ii) $\{y \in \mathbb{R}: y \geqslant 0\}$ (c) (i) $\{x \in \mathbb{R}: x \neq 0\}$ (ii) $\{y \in \mathbb{R}: y \neq 0\}$
(d) (i) $\{x \in \mathbb{R}: x \neq 3\}$ (ii) $\{y \in \mathbb{R}: y \neq 0\}$

Graphs

1. If $A = \{3, 4, 5, 6\}$ and $B = \{1, 2, 3, 4\}$, write down all the ordered pairs (x, y) such that $x \in A$, $y \in B$ and x is twice y.

2. If $A = \{2, 3, 4\}$, write down all the ordered pairs (x, y) such that $x \in A$, $y \in A$ and x is greater than y.

3. If $A = \{-2, -1, 0, 1, 2\}$ and $B = \{0, 1, 2, 3, 4\}$, write down all the ordered pairs (x, y) such that $x \in A$, $y \in B$ and $y = x^2$.

4. Three elements of the cartesian product $A \times B$ are (2, 3), (2, 4) and (3, 5). If there are six such ordered pairs in the cartesian product, find

(a) the sets A and B,
(b) the other three elements of $A \times B$,
(c) set C, a subset of $A \times B$, such that $C = \{(x, y): x \in A, y \in B \text{ and } x = y\}$.

5. If $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5, 6\}$, find the ordered pairs of set C given that $C = \{(x, y): x \in A, y \in B \text{ and } y = 2x\}$.

6. State which of the following points lie on the line $y = 8 - 3x$, (2, 2), $(-1, 5)$, (1, 5), $(4, -4)$.

7. If all of the following points lie on the line $y = 2x - 6$, find the values of a, b, c, d and e: $(5, a)$, $(2, b)$, $(-2, c)$, $(d, 2)$, $(e, 8)$.

8. If the point (2, 2) lies on the line $y = ax - 4$, find the value of a.

9. If the points (2, 1) and $(-2, -11)$ lie on the line $y = ax + b$, find the values of a and b.

10. Find where the following lines cut (i) the y-axis (ii) the x-axis.

(a) $y = x - 4$ (b) $y = 2x - 4$ (c) $y = 12 - 2x$
(d) $y = \frac{1}{2}x + 3$ (e) $y + 2x = 8$ (f) $y + 5x = 3$
(g) $2y - 5x = 12$ (h) $y = x^2 - 3x + 2$ (i) $y = x^2 + x - 6$

For each of the functions in questions **11** to **20**, (a) write down the equation of the function, (b) construct a table of values for the given domain, (c) plot the graph of the function for that domain.

11. $f: x \to x + 1$ for $|x| \leqslant 3$ **12.** $f: x \to x - 2$ for $|x| \leqslant 4$
13. $f: x \to 2x + 3$ for $|x| \leqslant 3$ **14.** $f: x \to x$ for $-2 \leqslant x \leqslant 4$

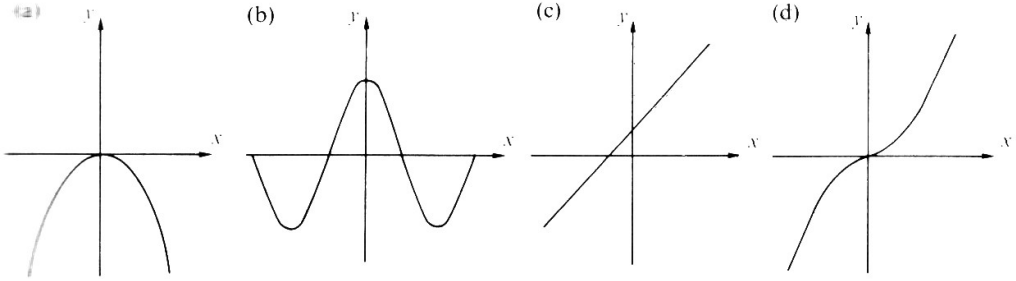
Answers

1. (4, 2), (6, 3) **2.** (3, 2), (4, 2), (4, 3) **3.** $(-2, 4)$, $(-1, 1)$, (0, 0), (1, 1), (2, 4)
4. (a) $A = \{2, 3\}$, $B = \{3, 4, 5\}$ (b) (2, 5), (3, 3), (3, 4) (c) $\{(3, 3)\}$ **5.** (1, 2), (2, 4), (3, 6)
6. (2, 2), (1, 5), $(4, -4)$ **7.** 4, -2, -10, 4, 7 **8.** 3 **9.** 3, -5
10. (a) (i) $(0, -4)$ (ii) (4, 0) (b) (i) $(0, -4)$ (ii) (2, 0) (c) (i) (0, 12) (ii) (6, 0)
(d) (i) (0, 3) (ii) $(-6, 0)$ (e) (i) (0, 8) (ii) (4, 0) (f) (i) (0, 3) (ii) $(\frac{3}{5}, 0)$
(g) (i) (0, 6) (ii) $(-2\frac{2}{3}, 0)$ (h) (i) (0, 2) (ii) (1, 0) and (2, 0)
(i) (i) $(0, -6)$ (ii) $(-3, 0)$ and (2, 0).

Parity

1. Show that each of the following functions are odd functions:
   (a) $f(x) = 7x$, (b) $f(x) = x^3 + x$, (c) $f(x) = 2x^3 - 3x$.
2. Show that each of the following functions are even functions:
   (a) $f(x) = 4x^2$, (b) $f(x) = 2 + x^2$, (c) $f(x) = 3x^2 + 2|x|$.
3. For each of the following functions, state whether they are even, odd or neither of these:
   (a) $f(x) = 4 - 3x^2$, (b) $f(x) = 3x^2 + x$, (c) $f(x) = x - \frac{1}{x}$,
   (d) $f(x) = x^2 + |x|$, (e) $f(x) = x^3 + |x|$.
4. For each of the following graphs, state whether they are graphs of odd functions, even functions or neither of these.

(a) (b) (c) (d)



5. Find the equations of the lines obtained if each of the following lines is reflected in the line $y = x$.
   (a) $y = \frac{x}{3}$, (b) $y = 4 - x$, (c) $y = 2x - 4$, (d) $y = \frac{1}{x + 2}$.

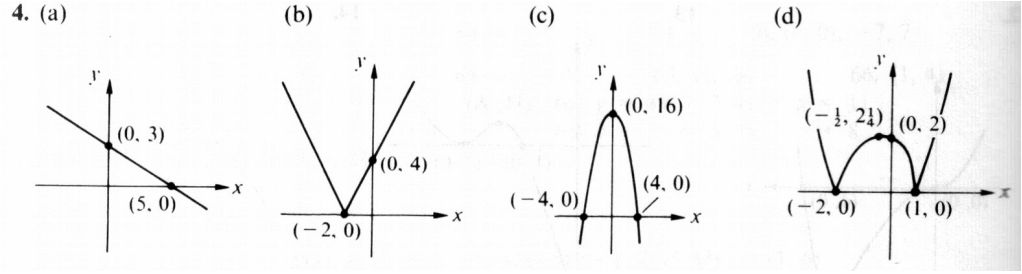
Answers

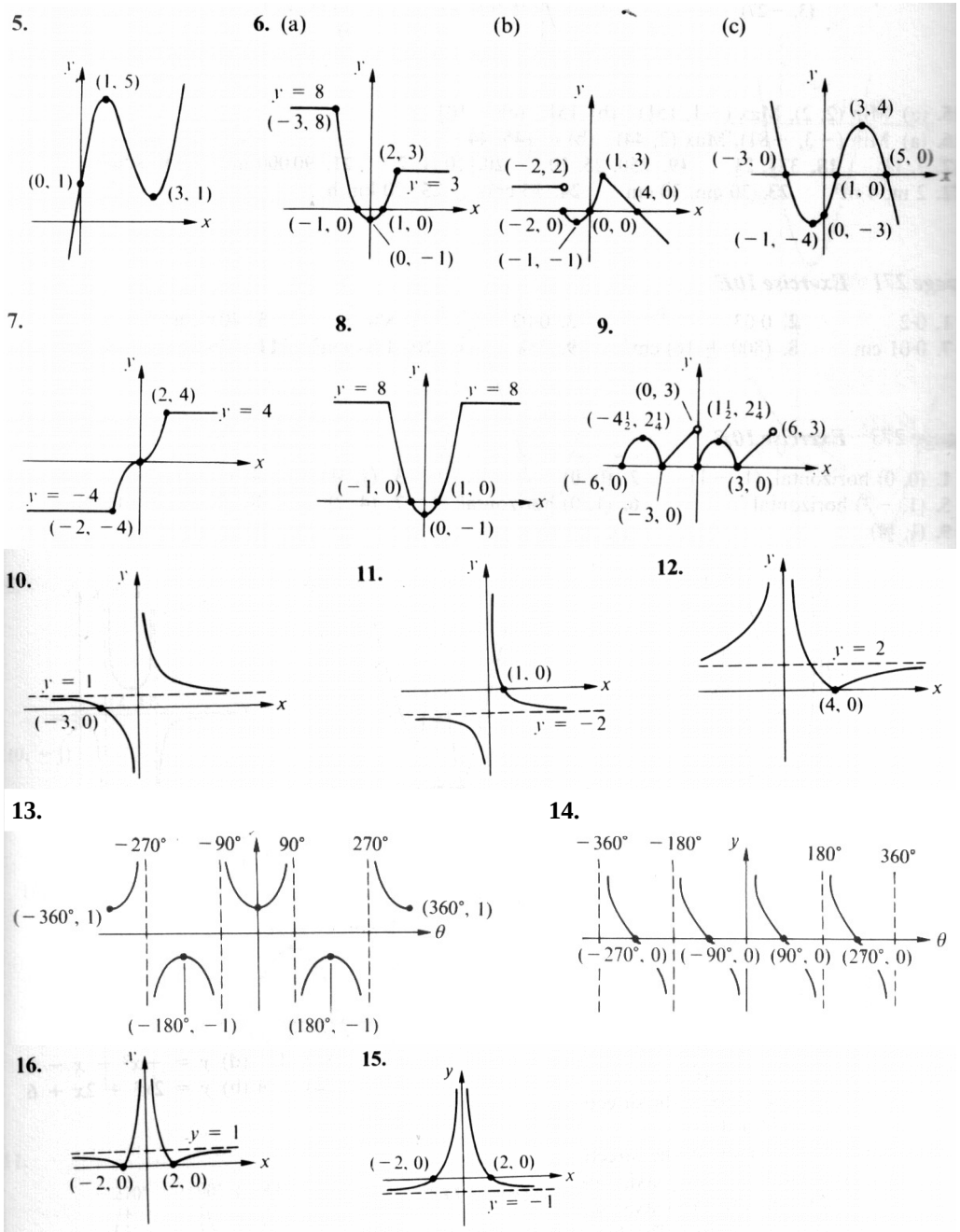
3. (a) even (b) neither (c) odd (d) even (e) neither
4. (a) even (b) even (c) neither (d) odd.
5. (a) $y = 3x$ (b) $y = 4 - x$ (c) $y = \frac{x + 4}{2}$ (d) $y = \frac{1}{x} - 2$

Boundedness and Extrema

1. Inspect boundedness and extrema for $f: y=|x|$
2. Inspect boundedness and extrema for $g: y=1-x^2$
3. Inspect boundedness and extrema for $f: y=\frac{|x|}{x}$

Find the global and local extrema of the function. State whether it is bounded above, below, both or neither.





Monotonicity and Periodicity

1. State the intervals of monotonicity for the functions in exercises 4 to 12 in the previous section.
2. Which of the functions are periodic? State the period.

Answers (Boundedness and Extrema)

1. Min at (0, 0), bounded below 2. Max at (0, 1), bounded above 3. Min = –1, Max = 1
4. (a) no extrema, unbounded (b) Min at (–2, 0), bounded below
   (c) Max at (0, 16), bounded above
   (d) local Max at (–½, 2¼), global Min (–2, 0) and at (1, 0), bounded below
5. local Max (1, 5) and local Min at (3, 1), unbounded
6. (a) global Max 8 for all $x \le -3$, global Min at (0, –1) local Max 3 for all $x \ge 2$, bounded
   (b) Max at (1, 3), local Min at (–1,–1), bounded above
   (c) local Max at (3, 4), local Min at (–1,–4), unbounded
7. Max 4 for all $x \ge 2$, Min –4 for all $x \le -2$
8. Max 8, Min at (0, –1)
9. local maximum 2¼ when $x = 1½ + 6k$ where k is integer, global minimum is 0 when x is a multiple of 3, bounded
10. no extrema, unbounded 11. no extrema, unbounded 12. Min at (4, 0), below
13. local maxima of –1 when x is an odd multiple of 180°, local minima of 1 when x is a multiple of 360°, unbounded
14. no extrema, unbounded 15. Min at (–2, 0) and (2, 0), below 16. bounded below

Answers (Monotonicity)

4. (a) decreasing (b) dec. in (–∞, –2), inc. in (–2, +∞)
   (c) inc. in (–∞, 0), dec. in (0, +∞) (d) inc. in (–∞, –2) and (–½, 1),
   dec. in (–2, –½) and (1,+∞)
5. inc. in (–∞, 1) and (3,+∞), dec. in (1, 3)
6. (a) const. in (–∞, –3) and (2,+∞), dec. in (–3, 0), inc. in (0, 2)
   (b) const. in (–∞,–2), dec. in (–2, –1) and (1,+∞), inc. in (–1, 1)
   (c) dec. in (–∞,–1) and (3,+∞), inc. in (–1, 3)
7. const. in (–∞, –2) and (2,+∞), inc. in (–2, 2)
8. const. in (–∞, –3) and (3,+∞), dec. in (–3, 0), inc. in (0, 3)
9. there are many such intervals, e.g. inc. in (–6, –4.5) and (–3, 0), dec. in (–4.5, 3)
10. and 11. dec. in (–∞, 0) and (0,+∞) 12. inc in (–∞, 0) and (4,+∞), dec. in (0, 4)

Answers (Periodicity)

9. period $p = 6$ 13. period $p = 360°$ 14. period $p = 180°$