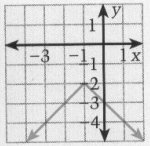
The Absolute Value Equations and Inequalities

1) The absolute value function

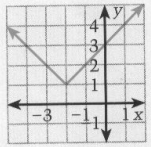
In Exercises 25–28, match the equation with its graph.

25. $y = |x - 1| + 2$ **26.** $y = -|x - 2| - 1$ **27.** $y = -|x + 1| - 2$ **28.** $y = |x + 2| + 1$

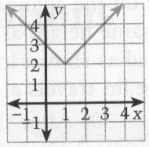
a.



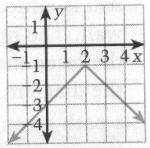
b.



c.

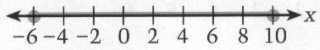


d.

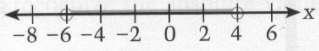


In Exercises 31–36, write an absolute value inequality to fit the graph.

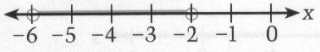
31.



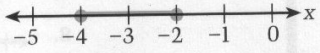
32.



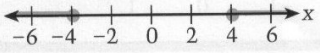
33.



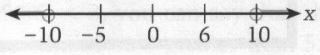
34.



35.



36.



2) Sketch the graph of the function:

37. $y = |x + 3|$ **38.** $y = |x - 5|$ **39.** $y = |x - 3|$ **40.** $y = |x + 4|$

41. $y = 2|x|$ **42.** $y = 3|x|$ **43.** $y = -2|x|$ **44.** $y = -\frac{1}{2}|x|$

45. $y = |x + 3| - 2$ **46.** $y = -|x - 1| + 4$

3) Graph two functions, one for each side , to solve the problem. Some need to be rearanged first so that you are able to sketch both graphs.

1) $|x| + 2x > 2$;
2) $|x + 2| + 1 > x$;
3) $|2x - 8| < 3x - 12$;
4) $|5x - 7| > 10x - 13$;

[1) $\left(\frac{2}{3}; +\infty\right)$; 2) R; 3) $(4; +\infty)$; 4) $\left(-\infty; \frac{4}{3}\right)$;

4) Graph two functions, one for each side , to solve the problem.Some need to be rearanged first so that you are able to sketch both graphs.

1) $|x + 19| = |x - 11|$;
2) $2|x - 5| = x$;
3) $|x - 3| = 1 - x$;
4) $|2x + 3| = 4 - x$;
5) $|x + 3| = 2x - 7$.

[1) -4; 2) $\frac{10}{3}$, 10; 3) $\emptyset$; 4) -7, $\frac{1}{3}$; 5) 10.]

5) Use a chart to solve the problem.

1) $|x| - |x - 1| = 2$;
2) $|x + 2| + |x - 1| = 3$;
3) $|x| + |x + 1| = 1$;
4) $|x + 1| + |x + 2| = 2$;
5) $|x - 1| - |x - 2| = 1$;
6) $|x - 2| + |x + 2| = 2x + 2$;
7) $2|x + 19| = |1 - x|$;
8) $4|x + \sqrt{2}| - 2|x - \sqrt{2}| = x$;
9) $|x - 3| + 3|x - 1| = 2x + 1$;
10) $|2x + 1| + |2x - 1| = 3$.

[1) $\emptyset$; 2) $\langle -2, 1\rangle$; 3) $\langle -1, 0\rangle$; 4) $-\frac{1}{2}$, $-\frac{5}{2}$; 5) $\langle 2, \infty\rangle$; 6) 1; 7) -39, $-\frac{37}{3}$; 8) $-2\sqrt{2}$, $\frac{-2\sqrt{2}}{5}$; 9) $\frac{5}{6}$, $\frac{7}{2}$; 10) $-\frac{3}{4}$, $\frac{3}{4}$.]

6) Use a chart to solve the problems in the given sets.

1) $|x - 2| + |x - 3| + |2x - 8| = 9$ in **R**
2) $|2x + 1| - |3 - x| = |x - 4|$ in **Z**
3) $|x - 1| + |1 - 2x| = 2|x|$ in **N**
4) $|x| - 2|x + 1| + 3|x + 2| = 0$ in [–3, 0)

[1) 1, $\frac{11}{2}$; 2) $\emptyset$; 3) 2; 4) -2.]

7) Use a chart to solve the problems.

1) $|x| < |x - 1|$;
2) $|x| \geqq |x - 1|$;
3) $|x + 2| \geqq |x - 2|$;
4) $|x + 5| < |2x - 1|$;
5) $|x - \frac{1}{2}| - |x + \frac{1}{2}| < 0$;
6) $3|x + 1| - |3x + 2| < 0$;

[1) $\left(-\infty; \frac{1}{2}\right)$; 2) $\left\langle \frac{1}{2}; +\infty\right)$; 3) $\langle 0; +\infty)$;
4) $\left(-\infty; -\frac{4}{3}\right) \cup (6; +\infty)$; 5) $(0, +\infty)$; 6) $\left(-\infty; -\frac{5}{6}\right)$;

8) Use a chart to solve the problems.

1) $10|x - 1| - 6|2 - x| + 2|x - 4| > 10 - 2x$;
2) $|2x + 6| + |3x - 12| + |x| < 20$;
3) $|x + 2| - 2|2x + 4| \leqq |3x - 1|$;

[1) $(-\infty; -1) \cup \left(\frac{3}{2}; +\infty\right)$; 2) $\left(-1; \frac{13}{3}\right)$; 3) $(-\infty; +\infty)$;

9) Use a chart to solve the inequalities like the ones with the absolute value.

1) $(x-3)(x+5) > 0$

2) $(2x-1)(x+2) \le 0$

3) $\frac{-3}{x+2} > 0$

4) $\frac{x-5}{2x+8} > 0$

5) $\frac{1}{2x} - \frac{1}{3x} > \frac{1}{4x} - 1$

6) $\frac{x+1}{x-2} > \frac{3}{x-2} - \frac{1}{2}$

7) $\frac{2}{x-1} < \frac{3}{x}$

8) $\frac{x-1}{x+2} > \frac{x+3}{x-2}$

9) $\frac{1}{x^2+4} > 0$

10) $\frac{(x+1)^2}{x^2+1} \le 0$

[**1**) $(-\infty,-5)\cup(3,\infty)$ **2**) $\left[2,\frac{1}{2}\right]$ **3**) $(-\infty,-2)$ **4**) $(-\infty,-4)\cup(5,\infty)$

5) $(-\infty,0)\cup\left(\frac{1}{12},\infty\right)$ **6**) $(-\infty,2)\cup(2,\infty)$ **7**) $(0,1)\cup(3,\infty)$

8) $(-\infty,-2)\cup\left(-\frac{1}{2},2\right)$ **9**) $\mathbb{R}$ **10**) $\{-1\}$]

10) Partition the given domain and then graph the function.

a) $f(x)=|x-3|-|x+3|-2 \quad D_f=[-5,3]$

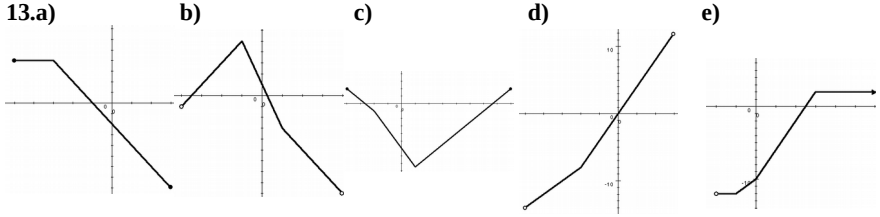
b) $f(x)=|1-x|-3|x+1|+3 \quad D_f=(-4,4)$

c) $f(x)=2|x-1|-0.5|x+2|-7 \quad D_f=[-4,8]$

d) $f(x)=|x+2|-|x-3|+2x+1 \quad D_f=(-5,3)$

e) $f(x)=|x+1|-2|x-3|+|x|-5 \quad D_f=(-5,\infty)$

13.a) **b)** **c)** **d)** **e)**



11) Solve the nested absolute value problems.

1) $||x+1|+|x-1||=2$

2) $||3-2x|-1|=2|x|$

3) $|x+|2x-5||>x+6$

4) $||x+2|-|x||\le|2x|$

[**1**) $[-1,1]$ **2**) $\left\{\frac{1}{2}\right\}$ **3**) $\left(-\infty,-\frac{1}{2}\right)\cup\left(\frac{11}{2},\infty\right)$ **4**) $(-\infty,-1)\cup[1,\infty)$]