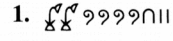
Numeral Systems

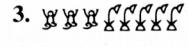
TABLE 4.1 Early Egyptian Symbols

Number	Symbol	Description
1		Stroke
10		Heel bone
100		Scroll
1,000		Lotus flower
10,000		Pointing finger
100,000		Burbot fish
1,000,000		Astonished person

Convert each Egyptian numeral to Hindu-Arabic form.

1. 

2.

3. 

4. 

Convert each Hindu-Arabic numeral to Egyptian form.

5. 427 **6.** 23,145 **7.** 306,090 **8.** 8,657,000

Chapter 1 of the book of Numbers in the Bible describes a census of the draft-eligible men of Israel after Moses led them out of Egypt into the Desert of Sinai, about 1450 B.C. Write an Egyptian numeral for the number of available men from each tribe listed.

9. 46,500 from the tribe of Reuben

10. 59,300 from the tribe of Simeon

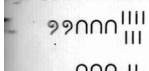
11. 45,650 from the tribe of Gad

12. 74,600 from the tribe of Judah

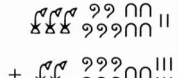
13. 54,400 from the tribe of Issachar

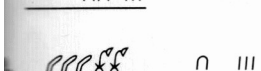
14. 62,700 from the tribe of Dan

Work each of the following addition or subtraction problems, using regrouping as necessary. Convert each answer to Hindu-Arabic form.

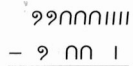
27. 

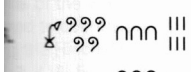
28. 

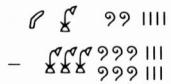
29. 

30. 

31. 

32. 

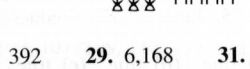
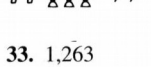
33. 

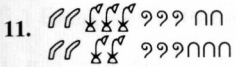
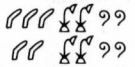
34. 

Use the Egyptian algorithm to find each product.

35. 3×19 **36.** 5×26 **37.** 12×93 **38.** 21×44

Selected Answers

1. 2,412 **3.** 3,005,231 **5.**  **7.**  **9.** 

11.  **13.**  **27.** 392 **29.** 6,168 **31.** 22 **33.** 1,263

35. 57 **37.** 1,116

Write each number in expanded form. (See Example 2.)

1. 37 **2.** 814 **3.** 2,815 **4.** 15,504

5. three thousand, six hundred twenty-eight

6. fifty-three thousand, eight hundred twelve

7. thirteen million, six hundred six thousand, ninety

8. one hundred twelve million, fourteen thousand, one hundred twelve

Simplify each of the following expansions. (See Example 3.)

9. $(7 \times 10^1) + (3 \times 10^0)$

10. $(2 \times 10^2) + (6 \times 10^1) + (0 \times 10^0)$

11. $(5 \times 10^3) + (0 \times 10^2) + (7 \times 10^1) + (2 \times 10^0)$

12. $(4 \times 10^5) + (0 \times 10^4) + (7 \times 10^3) + (7 \times 10^2) + (5 \times 10^1) + (2 \times 10^0)$

13. $(5 \times 10^7) + (6 \times 10^5) + (2 \times 10^3) + (3 \times 10^0)$

14. $(6 \times 10^8) + (5 \times 10^7) + (1 \times 10^2) + (4 \times 10^0)$

In each of the following, add in expanded notation. (See Example 4.)

15. $63 + 26$ **16.** $693 + 305$

In each of the following, subtract in expanded notation. (See Example 5.)

17. $84 - 52$ **18.** $673 - 412$

Perform each addition using expanded notation. (See Example 6.)

19. $65 + 44$ **20.** $536 + 279$ **21.** $424 + 298$ **22.** $6{,}755 + 4{,}827$

Perform each subtraction using expanded notation. (See Example 7.)

23. $53 - 47$ **24.** $253 - 48$ **25.** $643 - 436$ **26.** $826 - 345$

Compare the numeral systems below and for each of them state the base and whether it is a positional or a grouping system.

27. Roman numerals

1	I
5	V
10	X
50	L
100	C
500	D
1,000	M

28. Babylonian numeration

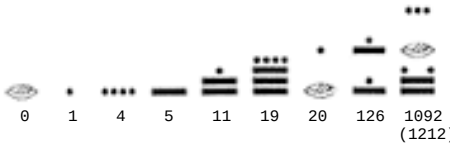
1	
10	

Example below:
$23 \times 60^1 + 41 \times 60^0 = 1421$

29. Greek numerals

1	α	60	ξ
2	β	70	ο
3	γ	80	π
4	δ	90	φ
5	ε	100	ρ
6	ς	200	σ
7	ζ	300	τ
8	η	400	υ
9	θ	500	φ
10	ι	600	χ
20	κ	700	ψ
30	λ	800	ω
40	μ	900	ϡ
50	ν		

30. Mayan numerals



Selected Answers

1. $(3\times10^1)+(7\times10^0)$ **3.** $(2\times10^3)+(8\times10^2)+(1\times10^1)+(5\times10^0)$ **5.** $(3\times10^3)+(6\times10^2)+(2\times10^1)+(8\times10^0)$
7. $(1\times10^7)+(3\times10^6)+(6\times10^5)+(0\times10^4)+(6\times10^3)+(0\times10^2)+(9\times10^1)+(0\times10^0)$ **9.** 73 **11.** 5,072
13. 50,602,003 **15.** 89 **17.** 32 **19.** 109 **21.** 722 **23.** 6 **25.** 207 **27.** *base 10, grouping*

Converting Between Number Bases

List the first twenty counting numbers in each of the following bases.

1. seven (Only digits 0 through 6 are used in base seven.)

2. eight (Only digits 0 through 7 are used.)

3. nine (Only digits 0 through 8 are used.)

4. sixteen (The digits 0, 1, 2, . . . , 9, A, B, C, D, E, F are used in base sixteen.)

For each of the following, write (in the same base) the counting numbers just before and just after the given number. (Do not convert to base ten.)

5. 14_{five} **6.** 555_{six} **7.** $B6F_{sixteen}$ **8.** $10{,}111_{two}$

Determine the number of distinct symbols needed in each of the following positional systems.

9. base three **10.** base seven **11.** base eleven **12.** base sixteen

Determine, in each of the following bases, the smallest and largest four-digit numbers and their decimal equivalents.

13. three **14.** sixteen

Convert each of the following to decimal form by expanding in powers and by using the calculator shortcut. (See Examples 1, 2, 5, 8, and 10.)

15. 24_{five} **16.** 62_{seven} **17.** $1{,}011_{two}$ **18.** 35_{eight}

19. $3BC_{sixteen}$ **20.** $34{,}432_{five}$ **21.** $2{,}366_{seven}$ **22.** $101{,}101{,}110_{two}$

23. $70{,}266_{eight}$ **24.** $A{,}BCD_{sixteen}$ **25.** $2{,}023_{four}$ **26.** $6{,}185_{nine}$

27. $41{,}533_{six}$ **28.** $88{,}703_{nine}$

Convert each of the following from decimal form to the given base. (See Examples 3, 4, 6, and 11.)

29. 86 to base five **30.** 65 to base seven **31.** 19 to base two

32. 935 to base eight **33.** 147 to base sixteen **34.** 2,730 to base sixteen

35. 36,401 to base five **36.** 70,893 to base seven **37.** 586 to base two

38. 12,888 to base eight **39.** 8,407 to base three **40.** 11,028 to base four

41. 9,346 to base six **42.** 99,999 to base nine

Make the following conversions as indicated. (See Example 7.)

43. 43_{five} to base seven **44.** 27_{eight} to base five

45. $C02_{sixteen}$ to base seven **46.** $6{,}748_{nine}$ to base four

Convert each of the following from octal form to binary form. (See Example 12.)

47. 367_{eight} **48.** $2{,}406_{eight}$

Convert each of the following from binary form to octal form. (See Example 13.)

49. $100{,}110{,}111_{two}$ **50.** $11{,}010{,}111{,}101_{two}$

Make the following conversions as indicated. (See Example 14.)

51. $DC_{sixteen}$ to binary **52.** $F{,}111_{sixteen}$ to binary

53. $101{,}101_{two}$ to hexadecimal **54.** $101{,}111{,}011{,}101{,}000_{two}$ to hexadecimal

Identify the largest number from each list in Exercises 55 and 56.

55. 42_{seven}, 37_{eight}, $1D_{sixteen}$ **56.** $1{,}101{,}110_{two}$, 407_{five}, $6F_{sixteen}$

One very common method of converting symbols into binary digits for computer processing is called ASCII. The upper case letters A *to* Z *are assigned the numbers* 65 *through* 90*, so* A *has binary code* 1000001 *and* Z *has code* 1011010*. Lowercase letters* a *through* z *have codes* 97 *through* 122 *(I.e.* 1100001 *through* 1111010*). ASCII codes, as well as other numerical computer output, normally appear without commas.*

Write the binary code for each of the following letters.

58. C 59. X 60. k 61. r

Break each of the following into groups of seven digits and write as letters.

62. 1001000100010110011001010000

63. 1000011100100010101011000011100101 1

Translate each word into an ASCII string of binary digits. (Distinguish upper and lower case.)

64. New 65. Orleans

66. *Add the numbers without conversion.*

a) $11{,}101_2 + 110{,}001_2$ d) $101{,}010_2 + 1{,}010_2$ g) $1{,}101_2 + 110_2 + 1{,}110_2$

b) $100{,}101_2 + 100{,}001_2$ e) $1{,}001_2 + 100_2 + 10_2$ h) $111{,}010_2 + 11{,}111_2 + 10_2$

c) $11{,}111_2 + 1{,}001{,}101_2$ f) $1{,}111_2 + 101_2 + 110_2$ i) $1{,}110{,}100_2 + 111{,}110_2 + 100_2$

67. *Add the numbers without conversion.*

a) $325_7 + 341_7$ c) $315_7 + 315_7$ e) $246_7 + 421_7$

b) $132_7 + 246_7$ d) $3{,}156_7 + 601_7$ f) $5{,}555_7 + 2{,}222_7$

68. *Multiply the binary numbers without conversion.*

a) 11 × 101 d) 1,001 × 1,001 g) 10,001 × 1,111

b) 1,011 × 100 e) 10,011 × 111 h) 10,111 × 11,001

c) 1,111 × 111 f) 11,011 × 1,010 i) 11,101 × 110,001

69. *Convert the binary numbers to hexadecimal.*

a) 110000101 c) 11111101 e) 1111000011110000

b) 100001 d) 1010101010 f) 1000001111011

Selected Answers

1. 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 26 **3.** 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22 **5.** 13_{five}; 20_{five} **7.** $B6E_{sixteen}$; $B70_{sixteen}$ **9.** 3 **11.** 11 **13.** smallest: $1{,}000_{three} = 27$; largest: $2{,}222_{three} = 80$ **15.** 14 **17.** 11 **19.** 956 **21.** 881 **23.** 28,854 **25.** 139 **27.** 5,601 **29.** 321_{five} **31.** $10{,}011_{two}$ **33.** $93_{sixteen}$ **35.** $2{,}131{,}101_{five}$ **37.** $1{,}001{,}001{,}010_{two}$ **39.** $102{,}112{,}101_{three}$ **41.** $111{,}134_{six}$ **43.** 32_{seven} **45.** $11{,}651_{seven}$ **47.** $11{,}110{,}111_{two}$ **49.** 467_{eight} **51.** $11{,}011{,}100_{two}$ **53.** $2D_{sixteen}$ **55.** 37_{eight} **57.** 1,427 **59.** 1011000 **61.** 1110010 **63.** CHUCK **65.** 100111111100101101100110010111000011101110110011

66. a) $100{,}110_2$ b) $1{,}000{,}110_2$ c) $1{,}101{,}100_2$ d) $110{,}100_2$ e) $1{,}111_2$ f) $11{,}010_2$ g) $100{,}001_2$ h) $1{,}011{,}011_2$ i) $10{,}110{,}110_2$

67. a) 666_7 b) 411_7 c) 633_7 d) $4{,}060_7$ e) $1{,}000_7$ f) $11{,}110_7$

68. a) $1{,}111_2$ b) $1{,}011{,}100_2$ c) $1{,}101{,}001_2$ d) $1{,}010{,}001_2$ e) $10{,}000{,}101_2$ f) $100{,}001{,}110_2$ g) $11{,}111{,}111_2$ h) $1{,}000{,}111{,}111_2$ i) $10{,}110{,}001{,}101_2$

69. a) 185_{16} b) 21_{16} c) FD_{16} d) $2AA_{16}$ e) $F0F0_{16}$ f) $107B_{16}$